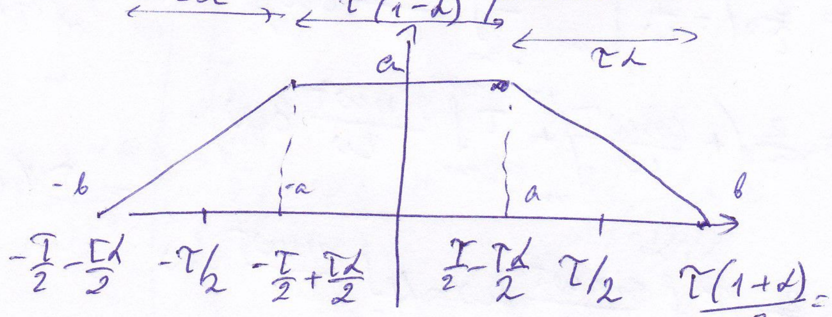
~~Влияние~~ формы импульса на ширину спектра $S(\omega)$. 5.5

$$\Delta t = \frac{\int_{-\infty}^{+\infty} |g(t)|^2 dt}{(\max|g(t)|)^2}$$ - длит. импульса $$\Delta\omega = \frac{\int_{-\infty}^{+\infty} |S(\omega)|^2 d\omega}{(\max|S(\omega)|)^2}$$

Найти $C = \Delta t \Delta\omega$ изобразив графически $g(t)$ и $S(\omega)$

$$g(t) = \begin{cases} a, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$

$g(t) = \{$ дольше переписывать, чем рисовать:



$$S(\omega) = +\frac{1}{2\pi}\int_{-\infty}^{+\infty} g(t) e^{-i\omega t} dt =$$

$$= \frac{1}{2\pi}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} a e^{-i\omega t} = \frac{a}{2\pi(-i\omega)} e^{-i\omega t}\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} =$$

$$= \frac{a}{2\pi(-i\omega)}\left(-2i\sin\frac{\omega\tau}{2}\right) = \frac{a}{\pi\omega}\sin\frac{\omega\tau}{2} =$$

$$= \frac{a\tau}{2\pi}\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} = \frac{a\tau}{2\pi}\operatorname{sinc}\frac{\omega\tau}{2}$$

$$\Delta t = \int_{-\infty}^{+\infty} |g(t)|^2 dt \Big/ (\max|g(t)|)^2 =$$

$\cancel{\omega \tau \frac{b}{\tau}}$

$$= \frac{a^2\tau}{a^2} = \tau$$ ~~при~~ ~~...~~

$$\Delta\omega = \int_{-\infty}^{+\infty} \frac{\left(\frac{a}{\pi\omega}\sin\frac{\omega\tau}{2}\right)^2 d\omega}{\left(\frac{a\tau}{2\pi}\right)^2} =$$

$$= \int_{-\infty}^{+\infty} \frac{4\sin^2\frac{\omega\tau}{2}}{\frac{\omega^2}{\tau^2}} d\omega = \frac{4}{\tau^2}$$

$$S(\omega) = \frac{1}{2\pi}\Bigg[\int_{-\frac{\tau(1+\alpha)}{2}}^{-\frac{\tau(1-\alpha)}{2}} \frac{a}{\alpha\tau}\left(t+\frac{\tau(1+\alpha)}{2}\right)e^{-i\omega t}dt$$

$$+\int_{-\frac{\tau(1-\alpha)}{2}}^{\frac{\tau(1-\alpha)}{2}} a e^{-i\omega t}dt + \int_{+\frac{\tau(1-\alpha)}{2}}^{\frac{\tau(1+\alpha)}{2}} -\frac{a}{\alpha\tau}\left(t-\frac{\tau(1+\alpha)}{2}\right)e^{-i\omega t}dt\Bigg]$$

$$= \left\{ \int t e^{ct} dt = t\frac{e^{ct}}{c} - \int \frac{e^{ct}}{c} dt = \right.$$

$$\left. = \frac{te^{ct}}{c} - \frac{e^{ct}}{c^2} + C \right\}$$

$$= \frac{2a}{\pi\tau\alpha\omega^2}\sin\frac{\omega\tau}{2}\sin\frac{\omega\alpha\tau}{2}$$

$$\Delta t = \int_{-\infty}^{+\infty}|g(t)|^2 dt \Big/ (\max|g(t)|)^2 =$$

$$= \left[a^2\tau(1-\alpha) + \int_a^b\left(\frac{a}{\alpha\tau}\left(t-\frac{\tau(1+\alpha)}{2}\right)\right)^2 dt + \right.$$

$$\left. + \int_{-b}^{-a}\left(\frac{a}{\alpha\tau}\left(t+\frac{\tau(1+\alpha)}{2}\right)\right)^2 dt\right] \Big/ a^2 =$$

$$= \left(a^2\tau(1-\alpha) + \frac{a^2}{\alpha^2\tau^2}\frac{\left(t-\frac{\tau(1+\alpha)}{2}\right)^3}{3}\Bigg|_a^b + \right.$$

$$\left. + \frac{a^2}{\alpha^2\tau^2}\frac{\left(t+\frac{\tau(1+\alpha)}{2}\right)^3}{3}\Bigg|_{-b}^{-a}\right) \Big/ a^2 =$$

$$= \frac{4}{\tau^2} \int_{-\infty}^{+\infty} \frac{\sin^2 \frac{\omega\tau}{2}}{\omega^2} d\omega =$$

$$= \frac{4}{\tau^2} \int_{-\infty}^{\infty} \frac{1 - \cos 2\frac{\omega\tau}{2}}{2\omega^2} d\omega =$$

$$= \frac{2}{\tau^2} \int_{-\infty}^{\infty} \frac{1 - \cos\omega\tau}{\omega^2} d\omega =$$

$$= \frac{2}{\tau^2}\left(-\frac{1}{\omega}\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \frac{\cos\omega\tau}{\omega^2} d\omega\right) =$$

$$= \frac{2}{\tau^2}\left(+\frac{\cos\omega\tau}{\omega}\Big|_{-\infty}^{+\infty} + \tau\int_{-\infty}^{+\infty} \frac{\sin\omega\tau}{\tau\omega} d\omega\right) =$$

$$= \frac{2}{\tau^2}\, \tau \int_{-\infty}^{+\infty} \frac{\sin\omega\tau}{\omega} d\omega = 4\int_{0}^{\infty} \frac{\sin\omega\tau}{\omega} d\omega$$

$$= \frac{4\, I(\tau)}{\tau}$$

$$= \left(a^2\tau(1-\alpha) + \frac{a^2}{\alpha^2\tau^2}\frac{\left(\cancel{a} - \frac{\tau(1+\alpha)}{2}\right)^3}{3}\right.$$

$$\left. + \frac{a^2}{\alpha^2\tau^2}\frac{\left(-a + \frac{\tau(1+\alpha)}{2}\right)^3}{3}\right) / a^2 =$$

$$= \cancel{a^2}\tau(1-\alpha) - \frac{1}{\alpha^2\tau^2}\frac{\left(\frac{\tau(1-\alpha)}{2} - \frac{\tau(1+\alpha)}{2}\right)^3}{3}$$

$$+ \frac{1}{\alpha^2\tau^2}\frac{\left(-\frac{\tau(1-\alpha)}{2} + \frac{\tau(1+\alpha)}{2}\right)^3}{3} =$$

$$= \tau(1-\alpha) - \frac{1}{\alpha^2\tau^2}\frac{(-\tau\alpha)^3}{3} + \frac{1}{\alpha^2\tau^2}\frac{(\tau\alpha)^3}{3} =$$

$$= \tau(1-\alpha) + \frac{2}{3}\tau\alpha = \tau - \tau\alpha + \frac{2}{3}\tau\alpha =$$

$= \tau - \frac{\tau\alpha}{3}$ – длина ~~сп~~ сигнала

$\Delta\omega = \{ \max S(\omega) = \{\omega = 0$ из графика$\} = \frac{2a}{\pi\tau\alpha} \cancel{\frac{2}{4}} \cdot \frac{\tau}{2}\cdot\frac{\alpha\tau}{2} = \frac{a\tau}{2\pi} \}$

$$= \frac{\int_{-\infty}^{+\infty}\left(\frac{2a}{\pi\tau\alpha\omega^2}\sin\frac{\omega\tau}{2}\sin\frac{\omega\tau\alpha}{2}\right)^2 d\omega}{\cancel{\left(\frac{2a}{\pi\tau\alpha}\right)^2}\left(\frac{a\tau}{2\pi}\right)^2} =$$

$$= \frac{16}{\tau^4\alpha^2}\int_{-\infty}^{+\infty}\left(\frac{\sin\frac{\omega\tau}{2}\sin\frac{\omega\tau\alpha}{2}}{\omega^2}\right)^2 d\omega$$

$$= \frac{4}{\tau^4\alpha^2}\int_{-\infty}^{+\infty}\frac{(1-\cos\omega\tau)(1-\cos\omega\tau\alpha)}{\omega^4} d\omega$$

$$= \frac{4}{\tau^4\alpha^2}\int_{-\infty}^{+\infty} -\frac{\cos\omega\tau}{\omega^4} - \frac{\cos\omega\tau\alpha}{\omega^4} + \frac{\cos\omega\tau\cos\omega\tau\alpha}{\omega^4} d\omega$$

$$\frac{1}{2\pi}\Big[\frac{a}{\alpha\tau}\Big(\frac{te^{-i\omega t}}{-i\omega} + \cancel{\frac{\tau(1+\alpha)}{2}} - \frac{e^{-i\omega t}}{(-i\omega)^2} + \frac{\tau(1+\alpha)}{2(-i\omega)}e^{-i\omega t}\Big|_{-\frac{\tau(1+\alpha)}{2}}^{-\frac{\tau(1-\alpha)}{2} = -a_0}$$

$$"-b"$$

$$+\left(-\frac{a}{\alpha\tau}\right)\left(\frac{te^{-i\omega t}}{-i\omega} - \frac{e^{-i\omega t}}{(-i\omega)^2} - \frac{\tau(1+\alpha)}{2(-i\omega)}e^{-i\omega t}\Big|_{\frac{\tau(1-\alpha)}{2}}^{\frac{\tau(1+\alpha)}{2} = b}\right)$$

$$"a"$$

$$+\frac{a}{-i\omega}e^{-i\omega t}\Big|_{-\frac{\tau(1-\alpha)}{2}}^{\frac{\tau(1-\alpha)}{2}}\Big] = \frac{1}{2\pi}\Big[\cancel{\frac{a}{\alpha\tau}}$$

$$\frac{a}{\alpha\tau}\Big[\frac{-a_0e^{+i\omega a}}{-i\omega} - \frac{(-b)e^{+i\omega b}}{-i\omega} - \frac{e^{i\omega a}}{(-i\omega)^2} + \frac{e^{i\omega b}}{(-i\omega)^2} + \frac{\tau(1+\alpha)}{2(-i\omega)}e^{i\omega a_0} - \frac{\tau(1+\alpha)}{2(-i\omega)}e^{i\omega b}$$

$$-\Big(\frac{be^{-i\omega b}}{-i\omega} - \frac{ae^{-i\omega a}}{-i\omega} - \frac{e^{-i\omega b}}{(-i\omega)^2} + \frac{e^{-i\omega a}}{(-i\omega)^2} - \frac{\tau(1+\alpha)}{2(-i\omega)}e^{-i\omega b} + \frac{\tau(1+\alpha)}{2(-i\omega)}e^{-i\omega a}\Big)\Big]$$

$$+\frac{a}{-i\omega}\left(-2i\sin\left(\omega\tau\left(\frac{1-\alpha}{2}\right)\right)\right)\Big] = \frac{1}{2\pi}\Big[\ ..$$

$$\frac{a}{\alpha\tau}\Big(\frac{a}{-i\omega}\left(-e^{i\omega a} + e^{-i\omega a}\right) + \frac{1}{(-i\omega)^2}\left(-e^{i\omega a} - e^{-i\omega a}\right) + \frac{1}{(-i\omega)^2}\left(e^{i\omega b} + e^{i\omega b}\right)$$

$$+\frac{\tau(1+\alpha)}{2(-i\omega)}\left(e^{i\omega a} - e^{-i\omega a}\right)\Big) + \frac{a}{\omega}2\sin\left(\omega\tau\frac{1-\alpha}{2}\right)\Big] =$$

$$\frac{1}{2\pi}\Big[\frac{a}{\alpha\tau}\Big(\frac{\tau(1-\alpha)}{2(-i\omega)}\cancel{\left(e^{i\omega a} + e^{-i\omega a}\right)}(-2i\sin\omega a) + \frac{1}{(-i\omega)^2}(-2\cos\omega a) +$$

$$+\frac{1}{(-i\omega)^2}2\cos\omega b + \frac{\tau(1+\alpha)}{2(-i\omega)}2i\sin\omega a\Big) + \frac{2a}{\omega}\sin\left(\omega\tau\frac{1-\alpha}{2}\right)\Big] =$$

$$\frac{1}{2\pi}\Big[\frac{a}{\alpha\tau}\Big(2\frac{\tau\cdot\alpha}{2(-i\omega)}\cdot 2i\sin\omega a + \frac{1}{\omega^2}2(\cos\omega a - \cos\omega b)\Big) + \frac{2a}{\omega}\sin\left(\omega\tau\frac{1-\alpha}{2}\right)\Big] =$$

$$= \frac{1}{2\pi}\Big[\frac{2a}{-\omega}\sin\omega\frac{\tau(1-\alpha)}{2} + \frac{a2}{\alpha\tau\omega^2}\left(\cos\omega\left(\frac{\tau(1-\alpha)}{2}\right) - \cos\omega\left(\frac{\tau(1+\alpha)}{2}\right)\right) + \frac{2a}{\omega}\sin\omega\tau\frac{1-\alpha}{2}\Big]$$

$$= \frac{1}{2\pi}\frac{2}{\omega^2}\frac{a}{\alpha\tau}\cdot 2\sin\frac{\omega\tau}{2}\cdot\sin\frac{\omega\alpha\tau}{2} = \frac{2a}{\pi\omega^2\tau\alpha}\sin\frac{\omega\tau}{2}\sin\frac{\omega\alpha\tau}{2} = S_f(\omega)$$

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$$\int_{-\infty}^{+\infty} \frac{\cos \omega c}{\omega^4} d\omega - \frac{\cos \omega c}{3\omega^3}\Bigg|_{-\infty}^{+\infty} - \frac{c}{3}\int_{-\infty}^{+\infty} \frac{\sin \omega c}{\omega^3} d\omega =$$

$$= -\frac{c}{3}\int_{-\infty}^{\infty} \frac{\sin c\omega}{\omega^3} d\omega = \frac{c}{3}\frac{\sin \omega c}{2\omega^2} - \frac{c^2}{6}\int_{-\infty} \frac{\cos \omega c}{\omega^2} d\omega =$$

$$= \frac{c^2}{6}\frac{\cos \omega c}{\omega} + \frac{c^3}{6}\int_{-\infty}^{+\infty} \frac{\sin \omega c}{\omega^2} d\omega \quad . \quad \int \frac{\cos \omega T\alpha}{\omega^4} d\omega \ \frac{\alpha^4}{\alpha}\int_{-\infty} \frac{\cos \omega T\alpha}{(\omega\alpha)^4} d\alpha\omega$$

$$\Rightarrow \Delta\omega = \frac{4}{T^4\alpha^2}\Bigg(-\frac{T^3}{6}\int_{-\infty}^{+\infty} \frac{\sin \omega T}{\omega} d\omega - \alpha^3 \cdot \frac{T^3}{6}\int_{-\infty} \frac{\sin \omega T}{\omega} d\omega +$$

$$+ \int_{-\infty}^{+\infty} \frac{\cos \omega T \cos \omega T\alpha}{\omega^4} d\omega\Bigg) =$$

$$= \frac{4}{T^4\alpha^2}\Bigg(-\frac{T^3}{6}(1+\alpha^3)\, 2I(T) + \underbrace{\int_{-\infty}^{+\infty} \frac{\cos \omega T \cos \omega T\alpha}{\omega^4} d\omega}_{\cancel{I_2(T)}}\Bigg)$$

$$\| \quad T^3\underbrace{\int_{-\infty}^{+\infty} \frac{\cos \omega \cos \omega\alpha}{\omega^4} d\omega}_{I_2}$$

$$\Rightarrow C_1 = T \cdot 4\frac{I_1(T)}{T} = 4I.$$

$$C_2 = \frac{4}{T^4\alpha^2}\Big(T^3 I_2 - I_1(T)\frac{T^3}{3}(1+\alpha^3)\Big)\cdot\Big(T - \frac{T\alpha}{3}\Big)$$

$$I = \int_0^\infty \frac{\sin \omega}{\omega} d\omega = \int_0^\infty \frac{\sin T\omega}{\omega T} d(\omega T) = \frac{T}{T}\int_0^\infty \frac{\sin T\omega}{\omega} d\omega$$

$$\cancel{\frac{\sin T\omega}{\omega} = T \cdot \frac{\sin T\omega}{\omega T}}$$

$$= \frac{4}{\alpha^2}\Big(I_2^{(\alpha)} - \frac{I_1}{3}(1+\alpha^3)\Big)\Big(1 - \frac{\alpha}{3}\Big)$$

$$\int_{-\infty}^{+\infty} \frac{\cos w \cos w\alpha}{w^4}\,dw = \int_{-\infty}^{+\infty} \frac{\frac{1}{2}(\cos(w - w\alpha) + \cos(w + w\alpha))}{w^4}\,dw =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \frac{\cos w(1-\alpha)}{w^4} + \frac{1}{2}\int_{-\infty}^{+\infty} \frac{\cos w(1+\alpha)}{w^4}\,dw = \{\text{уже считали}\} =$$

$$\#\left\{\int_{-\infty}^{+\infty} \frac{\cos wc}{w^4}\,dw = \frac{c^3}{6}2I\right\} = \frac{1}{2}\frac{(1-\alpha)^3}{6}2I + \frac{1}{2}\frac{(1+\alpha)^3}{6}2I =$$

$$= \frac{1 - 3\alpha + 3\alpha^2 - \alpha^3 + 1 + 3\alpha + 3\alpha^2 + \alpha^3}{12}2I = \frac{2(1+3\alpha^2)}{12}2I = \frac{1+3\alpha^2}{6}2I =$$

$$= \left(\frac{1}{6} + \frac{\alpha^2}{2}\right)I2 = \frac{1}{3} + \alpha^2 I$$

$$\Rightarrow C_2 = \frac{4}{\alpha^2}\left(\frac{1+3\alpha^2}{6}2I - \frac{I(1+\alpha^3)}{3}\right)\left(1 - \frac{\alpha}{3}\right) =$$

$$= \frac{4}{\alpha^2}\left(\frac{I}{3} + \alpha^2 I - \frac{I\alpha^3}{3} - \frac{I}{3}\right)\left(1 - \frac{\alpha}{3}\right) =$$

$$= \frac{4}{\alpha^2}\alpha^2\left(1 - \frac{\alpha}{3}\right)I\left(1 - \frac{\alpha}{3}\right) = 4\left(1 - \frac{\alpha}{3}\right)^2 I$$

При $\alpha = 0$ $C_1 = C_2$, и графики g_1 и g_2 совп.

$$I = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{\sin t}{t}\,dt = \left\{\frac{1}{t} = \int_0^{\infty} e^{-at}\,da\right\} = \int_0^{\infty} da \underbrace{\int_0^{\infty} e^{-at}\sin t\,dt}_{f(a)} = \left\{f(a) = \operatorname{Im}\int_0^{\infty} e^{-at+it}\,dt\right\}$$

$$= \operatorname{Im}\left.\frac{e^{-at+it}}{-a+i}\right|_0^{\infty} = \operatorname{Im}\frac{1}{-a+i} = \operatorname{Im}\frac{a+i}{a^2+1} = \frac{1}{a^2+1}\Big\} = \int_0^{\infty}\frac{da}{a^2+1} = \operatorname{arctg} a\Big|_0^{\infty} = \frac{\pi}{2}$$

$$\Rightarrow C_1 = 2\pi \qquad C_2 = 2\pi\left(1 - \frac{\alpha}{3}\right)^2$$